

Spin-isospin textured excitations in a double layer at filling factor

$$\nu = 2$$

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Abstract

We study the charged excitations of a double layer at filling factor 2 in the ferromagnetic regime. In a wide range of Zeeman and tunneling splittings we find that the low energy charged excitations are spin-isospin textures with the charge mostly located in one of the layers. As tunneling increases, the *parent* spin texture in one layer becomes larger and it induces, in the other layer, a *shadow* spin texture antiferromagnetically coupled to the *parent* texture. These new quasiparticles should be observable by measuring the strong dependence of its spin on tunneling and Zeeman couplings.

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Charged excitations of a single layer quantum Hall ferromagnet at filling factor $\nu = 1$ have been proposed to be textured quasiparticle excitations, *skyrmions* [1]. Skyrmions are relevant because at $\nu=1$ the system is an itinerant ferromagnet in which the topological charge associated with the spin texture is coupled to the actual electric charge [2]. The competition between the Zeeman splitting, Δ_z , and the Coulomb energy determines the size and energy of the skyrmions [3]. Different experimental techniques have confirmed that for typical values of Δ_z the skyrmions are the lowest energy charged excitations at $\nu=1$ [4–9]. Textured charged excitations could also exist in double layer (DL) systems at total filling factor $\nu=1$. In this case the layer index plays the role of an isospin, and the problem can be mapped onto that of an easy plane ferromagnet. Then, the finite energy isospin textured charged excitations are *bimerons* [10]. Alternatively, as we do here, it is possible to map the symmetric/antisymmetric states onto two isospin orientations.

The situation becomes rather more complicated when both spin and isospin must be considered simultaneously. This happens, for instance, for a DL at $\nu = 2$. This system presents a rich quantum phase diagram including ferromagnetic, canted antiferromagnetic and symmetric ground states (GS's) [11,12]. The existence and transitions between these phases have been recently observed [13]. In this system the spin and the isospin are strongly mixed, and a question to be raised is the possible existence of spin-isospin textured excitations (SITE) in DL systems at $\nu=2$. A serious difficulty in the study of charged excitations resides in the lack of an adequate field theory that could give insight on the kind of excitations to be expected.

In this Letter, we present a variational wave-function for describing the charge excitations in DL systems at $\nu = 2$ with interlayer separation d . In this work, we restrict to the regime of ferromagnetic GS. Our wave-function, which mixes spin and isospin textures, is built up in such a way that it recovers a skyrmion in one layer in the limit of zero tunneling splitting, Δ_{sas} . We have checked the adequacy of our wave-function by performing a numerical diagonalization for a few electrons system. We compute the energy, spin, isospin, and charge distribution of the charged excitation as a function of the DL parameters.

Our main results are:

i) The map of charged excitations, in terms of Δ_z and Δ_{sas} , presents two different regions as shown in Fig. 1. In one region, labeled as the one having single-particle excitations (SPE), the extra charge is shared by the two layers in a single-particle symmetric state with spin 1/2. In the other region, the excitation has a larger spin and the extra charge is mostly located in one of the layers. The transition between the two regions involves an abrupt change of both spin and isospin.

ii) In the second region, labeled as the one having a SITE, the charge induces in one layer a *parent* spin texture. Its spin is larger than that of the skyrmion in a single layer at $\nu = 1$ with the same Δ_z . This enlargement is accompanied by the appearance of a *shadow* spin texture in the other layer. *Parent* and *shadow* spin textures have an in-plane antiferromagnetic coupling. The existence of the *shadow* spin texture is associated with the existence of an isospin texture in the system so that the quasiparticle is a SITE.

Hamiltonian. We work in the lowest Landau level approximation where the single particle eigenstates in the symmetric gauge are labeled by the third component of the angular momentum m , the spin $\sigma = \uparrow, \downarrow (\pm 1)$ and the layer index $\Lambda = L, R$. The Hamiltonian of the DL is

$$H = -\frac{\Delta_{sas}}{2} \sum_{m\sigma} (c_{m\sigma L}^\dagger c_{m\sigma R} + \text{h.c.}) - \frac{\Delta_z}{2} \sum_{m\sigma\Lambda} \sigma c_{m\sigma\Lambda}^\dagger c_{m\sigma\Lambda} + \sum_{\{m_j\}\sigma\sigma'\Lambda\Lambda'} \frac{V^{\Lambda\Lambda'}_{m_1 m_2 m_3 m_4}}{2} c_{m_1\sigma\Lambda}^\dagger c_{m_2\sigma'\Lambda'}^\dagger c_{m_3\sigma'\Lambda'} c_{m_4\sigma\Lambda}. \quad (1)$$

The interaction potential is, $V^{\Lambda\Lambda} = e^2/(\epsilon r)$ and $V^{\Lambda\Lambda'} = e^2/(\epsilon \sqrt{r^2 + d^2})$ for $\Lambda \neq \Lambda'$. Here $r = \sqrt{x^2 + y^2}$ is the in-plane distance. Hereafter all distances will be given in units of the magnetic length l and the energies in units of $e^2/(\epsilon l)$.

GS at $\nu = 2$. The Hartree-Fock in-plane translational invariant solutions of this Hamiltonian have the form,

$$|\Psi_{GS}\rangle = \prod_{i=1}^2 \prod_{m=0}^{\infty} d_{mi}^\dagger |0\rangle. \quad (2)$$

Here $|0\rangle$ is the vacuum state and $d_{mi}^\dagger = \sum_{\sigma\Lambda} \alpha_{i\sigma\Lambda} c_{m\sigma\Lambda}^\dagger$ with the coefficients α determined to minimize the energy of the system. Note that i runs from 1 to 4, but since we work at $\nu = 2$, for each m we only fill up the two lowest energy states. As a function of the parameters d , Δ_z and Δ_{sas} , the GS given by Eq.(2) presents ferromagnetic, canted or singlet interlayer spin correlations [11,12]. In this Letter, we are interested in the regime of parameters giving a ferromagnetic GS at $\nu=2$. In this case, $d_{m1}^\dagger = c_{m\uparrow L}^\dagger$, $d_{m2}^\dagger = c_{m\uparrow R}^\dagger$, $d_{m3}^\dagger = c_{m\downarrow L}^\dagger$ and $d_{m4}^\dagger = c_{m\downarrow R}^\dagger$.

Charged excitations. We want to write down a wave-function describing the low energy charged excitations of the GS described by Eq.(2). This wave-function should have the freedom to get topological charge 1. The trial wave-function is allowed to get topological charge by mixing GS occupied orbitals with angular momentum m , ($d_{mj}^\dagger, j = 1, 2$) with GS empty orbitals of angular momentum $m + 1$ ($d_{m+1j}^\dagger, j = 3, 4$) [14]. With this, the form of our variational trial wave-function for the SITE has the circularly symmetric form

$$|\Psi_{qp}\rangle = \left(\gamma_3 d_{03}^\dagger + \gamma_4 d_{04}^\dagger \right) \times \prod_{i=1}^2 \prod_{m=0}^{\infty} \left(\sum_{j=1}^2 \beta_{ijm} d_{mj}^\dagger + \sum_{j=3}^4 \beta_{ijm} d_{m+1j}^\dagger \right) |0\rangle \quad (3)$$

The complex parameters β verify the constrains, $\sum_{j=1}^4 (\beta_{ijm})^* \beta_{i'jm} = \delta_{ii'}$. The first factor in Eq.(3) stands for the extra electron in $|\Psi_{qp}\rangle$ with respect to $|\Psi_{GS}\rangle$, and therefore $|\gamma_3|^2 + |\gamma_4|^2 = 1$. The complex parameters $\{\gamma\}$ and $\{\beta\}$ are obtained by minimizing the energy of the system while imposing that, as m increases, the coefficients β_{i3m} , β_{i4m} , β_{21m} and β_{12m} should decay to zero and β_{11m} and β_{22m} should tend to unity in order to recover the shape of the GS far from the center of the excitation.

Some comments about the wave-function (3) are in order:

First, we have checked the adequacy of allowing the mixing between states with angular momentum m and spin \uparrow in each layer ($c_{m\uparrow\Lambda}^\dagger$), and states with angular momentum $m + 1$ and spin \downarrow in any layer, ($c_{m+1\downarrow\Lambda'}^\dagger$). For this purpose, we have diagonalized numerically the Hamiltonian for 9 electrons in a double layer laterally confined by a parabolic potential. The potential has the adequate curvature to simulate a situation similar to the one we have in the thermodynamic limit [15].

Second, the wave-function (3) takes into account that the states belong to a four dimensional space corresponding to the spin and isospin degrees of freedom. This is the reason for the appearance of quartets in our trial wave-function for the SITE.

Third, in the limit of $d = 0$ and $\Delta_z = \Delta_{sas}$ our trial wave-function (3) is a good candidate to describe the skyrmions of the $SU(4)$ model [16].

Fourth, when the electron layers are decoupled ($\Delta_{sas}=0$), the GS is always ferromagnetic. If the extra electron is located in a single layer, (3) takes the form of the wave-function of a skyrmion in that layer at $\nu = 1$ [3].

Due to the form of $|\Psi_{qp}\rangle$ the minimization of the energy only requires diagonalization of 4×4 matrices, which are a generalization of the 2×2 matrices appearing for skyrmions in a single layer at $\nu = 1$ [3]. In general, the SITE is characterized by the existence of 10 independent order parameters. In our case of a ferromagnetic GS at $\nu=2$, the wave-function (3) involves broken spin and isospin symmetries as given by the order parameters

$$\langle c_{m\sigma\Lambda}^\dagger c_{m\sigma\Lambda'} \rangle \neq 0, \quad \langle c_{m\sigma\Lambda}^\dagger c_{(m+1)-\sigma\Lambda'} \rangle \neq 0. \quad (4)$$

From these order parameters we obtain the SITE energy, E , and the third components of both the SITE spin, $S = \sum_{m\sigma\Lambda} \langle c_{m\sigma\Lambda}^\dagger c_{m\sigma\Lambda} - 1 \rangle \sigma / 2$, and isospin, $I = \sum_{m\sigma} \langle c_{m\sigma R}^\dagger c_{m\sigma L} + c_{m\sigma L}^\dagger c_{m\sigma R} \rangle$.

Results. We have performed calculations in the whole regime of parameters giving a ferromagnetic GS at $\nu = 2$. The first important result of our Hartree-Fock calculation is the map of charged excitations, in terms of Δ_z and Δ_{sas} , shown in Fig. 1 for a representative case $d = l$. In the SPE region, the quasiparticle is not bounded to any texture and the extra charge is shared by the two layers in a trivial symmetric SPE. The energy of this quasiparticle is $(\Delta_z - \Delta_{sas})/2$, its spin $S=0.5$, and its isospin $I=1$. On the other hand, in the SITE region, we always find that $\gamma_3 \simeq 1$ and $\gamma_4 \simeq 0$. The extra charge is mostly located in one layer (here chosen as the left layer) while the charge located at the right layer is less than 0.05 electrons for all the cases in the SITE region. The existence of two different types of charged excitations also comes out from the numerical diagonalization for 9 electrons that

we have performed as mentioned above.

The transition between SITE and SPE is abrupt as shown in Fig 2. The curves start at the Δ_z values in which the GS becomes ferromagnetic. For low Δ_z the spin of the SITE is large. When Δ_z increases, the higher cost in Zeeman energy implies a reduction of the size and spin of the quasiparticle. For a given value of Δ_z , this energy cost is so large that the system changes abruptly to a SPE symmetric state with the spin having the lowest possible value (1/2). A very interesting result is the distribution of the SITE spin in the two layers. $S_R = \sum_{m\sigma} \langle c_{m\sigma R}^\dagger c_{m\sigma R} - 1 \rangle \sigma / 2$, the part of the SITE spin located at the right layer, is rather small as shown in Fig. 2. This reflects that the spin of the SITE is mainly due to the *parent* texture in the left layer.

The dependence of the SITE on tunneling is given in Fig. 3. S is shown as a function of Δ_{sas} . All curves are cutted off at the values of Δ_{sas} where the GS changes from ferromagnetic to canted antiferromagnetic [11,12]. At $\Delta_{sas} = 0$, the layers are decoupled and the spin texture is completely localized in the left layer while the right layer is inert. The excitation is exactly equal to a skyrmion for the same Δ_z in an isolated layer at $\nu = 1$ [3]. When tunneling is switched on (i. e. $\Delta_{sas} \neq 0$) S increases up to a value which depends on d .

In order to get some insight on the internal structure of the SITE, we compute the spin in-plane components on each layer $(S_x + iS_y)_\Lambda$. As sketched in the inset of Fig. 1, in both layers there is a winding number unity around the origin. Moreover, the *shadow* texture is dephased in π with respect to the *parent* texture. This agrees with the antiferromagnetic character of the effective interlayer coupling present in the quantum field theory description of the DL system [12].

The spin texture is accompanied by the appearance and increase of an isospin I given in Fig. 4. I increases significantly with tunneling. The dependence of I on Δ_z shows, once again, an abrupt change at the transition between SITE and SPE regions.

The physical picture coming out from all the above results is the following:

For small Zeeman and tunneling couplings, the system develops a single layer spin texture, as intralayer exchange is the most important interaction. When tunneling comes into

play, one could expect this single layer quasiparticle to eventually turn into a more symmetric spin texture, with the extra charge shared by the two layers. Our results show that this is not the case and the reason is drawn from the careful analysis of the energies competing in this problem. Exchange interaction plays a mayor role in keeping the extra charge mostly bound to one of the layers. Since the intralayer exchange is much stronger than the interlayer one, the system prefers primarily to form intralayer textures. However, a purely intralayer spin texture does not take any profit of the available tunneling energy. Our results show that, although the charge transfer from the left layer to the right one is very small, tunneling allows to move a great amount of charge from antisymmetric to symmetric states. A finite isospin appears and energy is gained preserving, at the same time, the quasi-intralayer character of the spin texture as preferred by exchange interaction. Moreover, the new local distribution of charge in the right layer, actually screens the *parent* texture in the left layer. The SITE becomes smoother and its spin increases, overcoming the spin corresponding to a purely single layer texture for the same Zeeman coupling. As Δ_z increases, the energy cost for creating an intralayer spin texture becomes eventually larger than the one corresponding to the trivial SPE. Then, the SITE shrinks abruptly into a SPE.

Experimental consequences. We have found that a bilayer system at $\nu = 2$ in the ferromagnetic regime can have charge excitations with a spin larger than $1/2$. This spin further increases with tunneling due to the formation of a SITE. Moreover, there is an abrupt change of the spin at the transition between the SITE and SPE regions of Fig. 1. The quasiparticle spin could be measured with the techniques used in establishing the existence of skyrmions in single layer at $\nu=1$: NMR [4], activation energy [5], optical absorption [6], specific heat [7]. In a particular sample, the Zeeman contribution could be varied either by applying a parallel magnetic field [5] or by changing the g -factor by means of external pressure [8,9]. In this way, the abrupt transition SITE-SPE could be detected. An alternative should be the study of a set of samples with different Δ_{sas} in order to analyze the increase of the SITE with tunneling.

In summary, we present a variational wave-function for the description of charged exci-

tations of a DL at $\nu = 2$ when the GS is ferromagnetic. In the map of charged excitations, there is a SITE region where, even for rather large tunneling, the extra charge is mostly located in one of the layers. The possibility of tunneling between layers provokes an increase of the spin texture compared with that of a skyrmion in a single layer at $\nu=1$ with the same Δ_z . This enlargement is accompanied by the appearance of an isospin texture. The characteristics of the SITE show a strong dependence on tunneling. The spin of the quasiparticle presents an abrupt transition when going from the SITE region to the SPE one by increasing Δ_z . These features should allow the experimental observation of these new quasiparticles.

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FIGURES

FIG. 1. Map of charged excitations in terms of Δ_z and Δ_{sas} (in units of $e^2/\varepsilon l$) for a representative case $d = l$. The shaded region corresponds to a non ferromagnetic GS for $\nu = 2$. SPE stands for single-particle excitations while SITE stands for the spin-isospin textured excitations schematically depicted at the inset. *Parent* and *shadow* spin textures are rotated in π with respect to each other.

FIG. 2. Spin S of the charged excitation as a function of Δ_z (in units of $e^2/\varepsilon l$) for $d = l$ and different values of Δ_{sas} . The part S_R of S due to electrons located at the right layer is also shown. The abrupt transition separates the region (to the left) corresponding to a SITE from that (to the right) corresponding to a SPE.

FIG. 3. SITE spin S as a function of Δ_{sas} (in units of $e^2/\varepsilon l$) for different values of Δ_z and d .

FIG. 4. Isospin I of the SITE as a function: a) of Δ_{sas} for $\Delta_z = 0.02$, and b) of Δ_z for $\Delta_{sas} = 0.1$. Δ_z and Δ_{sas} are in units of $e^2/\varepsilon l$. The abrupt transition in part b) separates the region (to the left) corresponding to a SITE from that (to the right) corresponding to a SPE.







